Review of EGN 216 - Circuits I: Steady-State Analysis

\[ R_L \quad KCL \quad KVL \]

\( R_L \), passivity rule: \( p > 0 \) absorbed, \( \sigma = Ri, p = \sigma \cdot i \)

Conductance: \( G = \frac{1}{R} \), siemens, S, mho, \( \Omega \)

**KCL**: \( \Sigma i = 0 \), gaussian surface, no accumulation of charge
conservation of matter
all exiting, all entering, exiting = entering

**KVL**: \( \Sigma \sigma = 0 \), conservation of energy, height analogy
all drops, all rises, drops = rises

Methods, rules, techniques, all derived from the tripod

\( R = 0 \Rightarrow \sigma = 0, \forall i : \text{short-circuit} \)

\( R = \infty \Rightarrow i = 0, \forall \sigma : \text{open-circuit} \)
Voltage division: $\frac{v_{R_i}}{R_i} = \frac{V}{\sum R}$

Series association: $R_S = \sum R$

Current division: $i_{R_i} = \frac{G_{i}}{\sum G} \cdot i$

special case: $\frac{G_i}{G_i + G_2} = \frac{R_2}{R_1 + R_2}$

Parallel association: $G_p = \sum G$

special case: $R_p = R_1//R_2 = \frac{1}{G_1 + G_2} = \frac{R_1 R_2}{R_1 + R_2}$

clue: $R_s > R_{\text{max}}$

$R_p < R_{\text{min}}$

not everything is series/parallel: $\Delta$-wye transformation
**Nodal Analysis**: KCL, looking for voltages

0. assign a reference node (\(V_0\))
1. identify and label the nodes
2. apply KCL to each node, using node voltages as variables
3. additional variables/substitutions as needed
4. solve for the voltages in the system of equations
5. return to the problem/circuit and obtain the answer(s)
6. verify...

**Direct matrix approach (by inspection)**: \(G \sigma = i\)

**Loop Analysis**: KVL, looking for currents

Special case: mesh analysis
1. identify and label circulating "mesh" currents
2. apply KVL to each mesh, using mesh currents as variables
3. additional variables/substitutions as needed
4. solve for the currents in the system of equations
5. return to the problem/circuit and obtain the answer(s)
6. verify...

**Direct matrix approach (by inspection)**: \(R i = \sigma\)
Linearity: \[ y = a_1 x_1 + a_2 x_2 + \ldots \]

- Proportionality (scaling)

- Superposition

\[ \text{Thévenin's Theorem (France, 1883)} \]

\[ V = V_{oc} - R_{th} i \]

\[ R_{th} = \frac{V_{oc}}{I_{sc}} \]

\[ y = ax + b \]

\[ \text{Norton's Theorem (Bell Labs, 1926)} \]

\[ i = I_{sc} - \frac{V}{R_{th}} \]

Source Transformation: Thévenin ↔ Norton
Obtaining the equivalent

(another point on the graph)

Thevenin resistance by inspection:  
- deachivate (kill) all sources  
- find equivalent wrt terminals  
- only works with independent sources

Thevenin-based technique

1. remove circuit element of interest  
2. obtain the thevenin equivalent of what is left  
3. reinsert the circuit element and obtain the answer (trivial)

Maximum Power Transfer Theorem

max power:  \( R_L = R_{th} \)
Capacitor: \[ q = CV \]
\[ C: \text{capacitance, farad, } F \]
\[ \frac{dq}{dt} = i = C \frac{dv}{dt}, \quad C_p = \Sigma C \]
\[ E_c = \frac{1}{2} CV^2 \]

Inductor: coil, magnetic field (more in EGN323)
\[ L: \text{inductance, henry, } H \]
\[ \varphi = L \frac{di}{dt}, \quad L_s = \Sigma L \]
\[ E_L = \frac{1}{2} LI^2 \]
Analogy to mechanical elements:  

\[ i \rightarrow f, \ e \rightarrow \sigma \text{ model} \]

 mechanical electrical component equivalency

\[ f = ma = m \frac{dv}{dt} \quad i = C \frac{de}{dt} \quad m = C \]

\[ f = kx \Rightarrow \frac{df}{dt} = kv \quad e = L \frac{di}{dt} \quad \frac{1}{k} = L \]

\[ f = bv \quad e = Ri \quad \frac{1}{b} = R \text{ or } b = G \]

\[ i \rightarrow \sigma, \ e \rightarrow f \text{ model} \]

 mechanical electrical component equivalency

\[ f = m \frac{dv}{dt} \quad e = L \frac{di}{dt} \quad m = L \]

\[ \frac{df}{dt} = kv \quad i = C \frac{de}{dt} \quad \frac{1}{k} = C \]

\[ f = bv \quad e = Ri \quad b = R \]

Energy:

\[ E_m = \frac{1}{2} mv^2, \quad E_k = \frac{1}{2} kx^2 = \frac{1}{2k} f^2 \]
Motivation of 1st order systems

- electrical:

\[ R i(t) + L \frac{di(t)}{dt} = e(t) \]

\[ \frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{1}{L} e(t) , \quad \tau = \frac{L}{R} \]

- mechanical:

\[ f(t) = f_i(t) - f_d = m \ddot{x} = m \frac{d\omega(t)}{dt} \]

\[ f_d = b \dot{x}(t) \]

\[ :\quad m \frac{d\omega(t)}{dt} + b \omega(t) = f_i(t) \]

\[ \frac{d\omega(t)}{dt} + \frac{b}{m} \omega(t) = \frac{1}{m} f_i(t) , \quad \tau = \frac{m}{b} \]

\[ \dot{\omega} + \frac{b}{m} \omega = \frac{1}{m} f_i(t) \]
Fluid:

\[ q_{mi} - q_{mo} = \frac{dm(t)}{dt} = C_f \frac{dp(t)}{dt} \]

\[ C_f \frac{dp(t)}{dt} + \frac{1}{R_f} p(t) = q_{mi}(t) \]

\[ \therefore p + \frac{1}{R_f C_f} p = \frac{1}{C_f} q_{mi}(t), \quad \zeta = R_f C_f \]

Electrical analog:

\[ q_{mi}(t) \]

\[ p(t) \]

Thermal:

\[ E_h = C_h T \Rightarrow q_h = \frac{dE_h}{dt} = C_h \frac{dT}{dt} \]

Newton: \( \Delta T = R_h q_h \)

\[ q_h = C_h \frac{dT}{dt} = \frac{T_b - T}{R_h} \]

\[ C_h \frac{dT(t)}{dt} + \frac{1}{R_h} T(t) = \frac{1}{R_h} T_b(t) \]

\[ T + \frac{1}{R_h C_h} T = \frac{1}{R_h C_h} T_b(t), \quad \zeta = R_h C_h \]
Complex Algebra

\[ z = x + jy = z e^{j\theta} = z |\theta| \]

**Euler identity:** \( e^{j\theta} = \cos \theta + j\sin \theta \)

**Addition, multiplication:** rules, practice

**Linear operators:** \( \text{Re}[z] = x, \text{Im}[z] = y \)

\[ j = e^{j90^\circ}, \quad \frac{1}{j} = -j \]

**Sinusoids:** \( x(t) = X_m \cos(\omega t + \Theta) \)

- \( X_m \): maximum value, peak value, amplitude
- \( \Theta \): phase (radians)
- \( \omega \): angular frequency (rad/s)
- \( f \): frequency, hertz, Hz
- \( T \): period (s)

\[ f = \frac{1}{T}, \quad \omega = 2\pi f \]
**Phasors**

\[ \phi(t) = V_m \cos(\omega t + \theta) = \text{Re} \left[ V_m e^{j(\omega t + \theta)} \right] = \text{Re} \left[ e^{j\omega t} (V_m e^{j\theta}) \right] \]

\[ \tilde{V} = V_m e^{j\theta} = V_m L \theta \]: phasor, having magnitude \( V_m \) and phase \( \theta \), a complex number applicable to sinusoidal voltages and currents

**Circuit elements**

Resistor: \[ \phi(t) = RI(t) \Rightarrow \tilde{V} = R \tilde{I} \] (voltage and current in phase)

Inductor: \[ \phi(t) = L \frac{di(t)}{dt} \Rightarrow \tilde{V} = j\omega L \tilde{I} \] (voltage 90° ahead of current)

Capacitor: \[ i(t) = C \frac{dv(t)}{dt} \Rightarrow \tilde{I} = j\omega C \tilde{V} \] (current 90° ahead of voltage)

**Impedance** (generalized resistance)

\[ \hat{Z} \triangleq \frac{\tilde{V}}{\tilde{I}} = \frac{V_m}{I_m} \frac{L \theta_b - \theta_i}{I_m} = R + jX \]

- \( R \): resistance
- \( X \): reactance
- \( \hat{Z} \): impedance

\[ \hat{Z}_R = R, \quad \hat{Z}_L = j\omega L, \quad \hat{Z}_C = \frac{1}{j\omega C} = -\frac{1}{\omega C} \]

\[ X_L = \omega L, \quad X_C = \frac{1}{\omega C} \]

\[ \hat{Z}_L = jX_L, \quad \hat{Z}_C = -jX_C \] (different from the book!!)
Admittance

\[ \hat{y} = \frac{1}{\hat{Z}} = \frac{I}{\hat{V}} = G + jB \]

\(G:\) conductance  
\(B:\) susceptance  
\(\hat{y}:\) admittance

\[ \hat{y}_R = G = \frac{1}{R} , \quad y_L = \frac{1}{j\omega L} = \frac{-j}{\omega L} , \quad y_C = j\omega C \]

\[ B_L = \frac{1}{\omega L} \quad B_C = \omega C \]

\[ \hat{y}_L = -jB_L \quad \hat{y}_C = jB_C \]

All laws and circuit analysis techniques hold true in the phasor domain. Impedance and admittance are not phasors!!
Phasor diagrams

series: \( \hat{Z} = R + jX \)

\[ \begin{align*}
\hat{V} &= \hat{V}_R + \hat{V}_X = R\hat{I} + jX\hat{I} = \hat{Z}\hat{I} \\
\hat{V}_R &= \hat{V}_X \\
\hat{I} &= \hat{I}_R + \hat{I}_X \\
\end{align*} \]

parallel: \( \hat{Y} = G + jB \)

\[ \begin{align*}
\hat{I} &= \hat{I}_G + \hat{I}_B = \hat{G}\hat{V} + j\hat{B}\hat{V} = \hat{Y}\hat{V} \\
\hat{I}_G &= \hat{I}_B \\
\hat{V} &= \hat{V}_G + \hat{V}_B \\
\end{align*} \]