ELE 172: Digital Logic

Quiz 2

February 7, 2011

IMPORTANT: Open notes, open book. Show all work. Answers, even if correct, without evidence of how the answer was reached will not be credited. Duration: 15 minutes.

Problem 1. (1 point)

Simplify the following switching expression using postulates of Boolean algebra only. At each step identify the applicable postulate.

\[ f(a, b, c) = a \cdot \bar{b} \cdot \bar{c} + \bar{b} \cdot c + a \cdot \bar{b} \cdot c + a \cdot \bar{b} \cdot c + a \cdot b \cdot c \]

\[ = \bar{a} \cdot \bar{b} \cdot (\bar{c} + c) + \bar{b} \cdot c + a \cdot b \cdot (\bar{c} + c) \]

\[ = \bar{a} \cdot \bar{b} + \bar{b} \cdot c + a \cdot b \]

\[ = a \cdot (\bar{b} + b) + \bar{b} \cdot c \]

\[ = a + \bar{b} \cdot c \]

Diagram:

Problem 2. (2 points)

Simplify the expression shown below so that the only complemented terms are individual variables. Identify any Postulates or theorems you used in each transformation step.

\[ f(a, b, c) = \overline{(a \cdot \bar{b} + a \cdot \bar{b} \cdot \bar{c}) + c} \]

\[ = \overline{(ab + a/b/c) \cdot c} \]

\[ = \overline{(a/b) \cdot \bar{c}} \]

\[ = \overline{(a \cdot b)c} \]

\[ = a \cdot \bar{c} + b \cdot c \]

Diagram:
ELE 172: Digital Logic

Quiz 3

February 16, 2011

IMPORTANT: Open notes, open book. Show all work. Answers, even if correct, without evidence of how the answer was reached will not be credited. Duration: 20 minutes.

Problem 1. (2 points)

Given the switching function \( f(a, b, c) = \overline{a} \cdot \overline{b} + \overline{a} \cdot \overline{b} \cdot \overline{c} \) use algebra (De Morgan’s and expansion theorems) to determine the canonical SOP form, then convert to minterm list, and obtain the truth table.

\[
\begin{align*}
 f(a, b, c) &= \overline{(a \cdot b)} \cdot \overline{(a \cdot b \cdot c)} = (\overline{a+b}) (a+b+c) \\
 &= \overline{a} + \overline{a} \cdot \overline{b} + \overline{a} \cdot \overline{c} + \overline{a} \cdot \overline{b} + \overline{a} \cdot \overline{b} \cdot \overline{c} = \overline{a} + \overline{b} + \overline{c}
\end{align*}
\]

\[
\begin{align*}
 f_{\text{SOP}}(a, b, c) &= \overline{a} \cdot \overline{b} + \overline{a} \cdot \overline{b} \cdot \overline{c} + (\overline{a} + a) \cdot \overline{b} \cdot \overline{c} = \overline{a} \cdot \overline{b} \cdot \overline{c} + \overline{a} \cdot \overline{b} \cdot \overline{c} + \overline{a} \cdot \overline{b} \cdot \overline{c} + \overline{a} \cdot \overline{b} \cdot \overline{c} \cdot \overline{a} \cdot \overline{b} \cdot \overline{c} \\
 &= \sum (0, 1, 2, 3, 5) \quad \overline{a} \cdot \overline{b} \cdot \overline{c} + \overline{a} \cdot \overline{b} \cdot \overline{c} \cdot \overline{a} \cdot \overline{b} \cdot \overline{c}
\end{align*}
\]

Problem 2. (2 points)

\( \overline{\text{M}(1, 3, 7)} \)

Given the switching function \( f(a, b, c) = \sum (0, 2, 4, 5, 6) \) obtain the canonical SOP and POS forms of the function. Use algebra to obtain the minimum realization and draw the resulting circuit using NAND gates only.

\[
\begin{align*}
 f_{\text{SOP}}(a, b, c) &= (a \cdot \overline{b} \cdot \overline{c} + a \cdot \overline{b} \cdot c + a \cdot \overline{b} \cdot c + a \cdot \overline{b} \cdot c + a \cdot \overline{b} \cdot c) \\
 f_{\text{POS}}(a, b, c) &= (a + b + c)(a + \overline{b} + c)(a + b + c)
\end{align*}
\]

\[
\begin{align*}
 f(a, b, c) &= \overline{a} \cdot \overline{b} \cdot c + \overline{a} \cdot \overline{b} + \overline{a} \cdot c \\
 &= \overline{a} \cdot \overline{b} \cdot c + \overline{a} \cdot \overline{b} + \overline{a} \cdot c = \overline{a} \cdot \overline{b} \cdot c + \overline{a} \cdot \overline{b} = \overline{a} \cdot \overline{b} \cdot c + \overline{a} \cdot \overline{b}
\end{align*}
\]
ELE 172: Digital Logic

Quiz 4

March 2, 2011

IMPORTANT: Open notes, open book. Show all work. Answers, even if correct, without evidence of how the answer was reached will not be credited. Duration: 15 minutes.

Problem 1. (2 points)

Given the following 3-variable function $f(a, b, c) = \overline{ab + bc + \overline{abc}}$

(a) Obtain the K-map representation of $f(a, b, c)$.

(b) Find the minimum SOP realization of $f(a, b, c)$.

\[ \text{SOP for } f(a, b, c) = \text{\overline{a \cdot \overline{b + c}}} \]

\[ \text{POS for } f(a, b, c) = \text{\overline{a \cdot (b + \overline{c})}} \]

Problem 2. (1 point)

Find the minimum realization for $f(a, b, c, d) = \Sigma m(5, 6, 7, 8, 12, 13, 15) + \Sigma d(0, 1, 2, 10)$. Use the following K-map:

\[ f = bd + \overline{bd} + ab \overline{c} + \overline{abc} \]

\[ f = bd + \overline{a} \overline{c} \overline{d} + \overline{abc} \]

\[ f = (a + c + d)(\overline{a + c} + d)(b + \overline{d}) \]
**ELE 172: Digital Logic**

**Quiz 6**

March 16, 2011

**IMPORTANT:** Open notes, open book. Show all work. Answers, even if correct, without evidence of how the answer was reached will not be credited. Duration: 15 minutes.

- **Problem 1. (2 points)**

  Given the logic function $f(a, b, c) = \sum m(1, 2, 3, 4, 6, 7)$ show realizations of this function using:
  
  (a) 2- and 3-input NAND gates only (i.e., 7400 and 7410).
  
  (b) 4-to-1 multiplexer (i.e., 74153).

- **Problem 2. (2 points)**

  Design a 3-input ($x_1$, $x_2$, $x_3$), 3-output ($y_1$, $y_2$, $y_3$) circuit such that returns the 2s complement of the input. For example $001 \rightarrow 111$.

\[
\begin{array}{c|c|c}
  x_2 & x_1 & x_0 \\
  \hline
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
  0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
  0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  \hline
  y_2 & y_1 & y_3 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
  x_2 & x_1 & x_0 & 0 & 1 & 1 & 0 & 1 \\
  \hline
  0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
  0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
  1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
  1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  \hline
  y_2 = /x_2 x_0 + /x_2 x_1 + x_2 /x_1 /x_0 \\
  y_1 = /x_1 x_0 + x_1 /x_0 \\
  y_0 = x_0
\end{array}
\]
ELE 172: Digital Logic

Quiz 6

April 4, 2011

IMPORTANT: Open notes, open book. Show all work. Answers, even if correct, without evidence of how the answer was reached will not be credited. Duration: 15 minutes.

Problem 1. (2 points)

Given the T-flip-flop circuit shown in Figure 1 obtain:

(a) Output and excitation equations.
(b) Complete the timing trace shown below.

\[ T = x \oplus y \]

\[ z = x \oplus y \]

Figure 1. Sequential circuit for Problem 1.

Problem 2. (2 points)

Obtain the state diagram and the next state equations given the following state table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Q_1 )</th>
<th>( Q_0 )</th>
<th>( Q_1' )</th>
<th>( Q_0' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>0 0</td>
<td>0 1</td>
<td>0 1</td>
<td>0 1</td>
<td></td>
</tr>
<tr>
<td>0 1</td>
<td>1 0</td>
<td>1 0</td>
<td>1 0</td>
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<tr>
<td>0 1</td>
<td>0 1</td>
<td>0 0</td>
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<td>1 0</td>
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<td>1 0</td>
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<tr>
<td>1 1</td>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>1 1</td>
<td>1 0</td>
<td>1 0</td>
<td>1 0</td>
<td></td>
</tr>
</tbody>
</table>

\[ Q_1 = Q_2 Q_1' + x \cdot Q_1 \]

\[ Q_0 = x Q_2 Q_1 + x \cdot Q_1 + \overline{x} \cdot Q_0 \cdot Q_1' \]
ELE 172: Digital Logic

Quiz 8

April 11, 2011

IMPORTANT: Open notes, open book. Show all work. Answers, even if correct, without evidence of how the answer was reached will not be credited. Duration: 15 minutes.

- Problem 1. (2 points)

Given the state and output table and state assignments shown below realize the circuit using JK flip-flops. (For partial credit, you may use D flip-flops.)

\[
\begin{array}{c|c|c|c}
A & 0 & 1 \\
\hline
A & A/1 & B/0 \\
B & C/0 & B/0 \\
C & B/1 & A/1 \\
\end{array}
\]

- Problem 2. (2 points)

Design a one-input, one-output Moore-type clocked sequential circuit that recognizes the input sequence 011. For example, the following input/output sequences describes the desired behavior:

\[
x = 010111011010 \\
z = 000001000100
\]

(a) Show the state diagram and the state table.
(b) Determine the number of flip-flops.
(c) Given the sequence of inputs \( x \) shown above, determine the resulting sequence of states (assume any convenient state as your initial state)
ELE 172: Digital Logic

Quiz 9

April 20, 2011

IMPORTANT: Open notes, open book. Show all work. Answers, even if correct, without evidence of how the answer was reached will not be credited. Duration: 15 minutes.

Problem 1. (2 points)

Given the state and output table shown below realize the circuit using T flip-flops. (Symbols x and y denote inputs.)

\[ T = \overline{Q_x} + Q_y \]

Problem 2. (2 points)

Show the design of one bit of an n-bit parallel-in, parallel-out register with LOAD control using T-type flip-flops.

\[ T = \overline{D_i}L_i + D_iL_i / Q_i \]