Problem 7.55

The voltage source in the circuit shown below is given as:

\[
    v(t) = \begin{cases} 
        12, & 0 \leq t \leq 1 \\
        0, & \text{otherwise} 
    \end{cases}
\]

Find the voltage \( v_0(t) \).

Solution

Node equation at the center node results in

\[
    \frac{v_1(t) - v(t)}{2k} + \frac{v_1(t)}{6k} + i_C(t) = 0 \tag{1}
\]

KVL around the right mesh gives us necessary auxiliary equation:

\[
    -v_1(t) + 2k i_C(t) + v_C(t) = 0 \tag{2}
\]

Substituting for the current, simplifying and rearranging terms we get the following two differential equations with two unknowns \( v_C(t) \) and \( v_1(t) \):

\[
    \frac{6}{5} \frac{d v_C(t)}{dt} + 3 v_1(t) = 2 v(t) \tag{3}
\]

\[
    \frac{2}{5} \frac{d v_C(t)}{dt} + v_C(t) - v_1(t) = 0 \tag{4}
\]

Solving for \( v_1(t) \) in Eq. (4) and substituting into (3) we get

\[
    \frac{12}{5} \frac{d v_C(t)}{dt} + 3 v_C(t) = v(t) \tag{5}
\]

\[
\text{In[7]} = \text{DSolve}\left[\left\{\frac{12}{5} v_C'[t] + 3 v_C[t] == 24 (\text{UnitStep}[t] - \text{UnitStep}[t - 1]), v_C[0] == 0\right\}, v_C[t], t\right] // \text{Simplify} // \text{First}
\]

\[
\text{Out[7]} = \left\{v_C[t] \rightarrow \begin{cases} 
8 e^{-5 t/4} (-1 + e^{5/4}) & t > 1 \\
8 - 8 e^{-5 t/4} & 0 \leq t \leq 1 \\
0 & \text{True}
\end{cases} \right\}
\]

Plot the voltage \( v_0(t) = v_C(t) \).