In a metal the various energy bands overlap to give a single band of energies that is only partially full of electrons. There are states with energies up to the vacuum level where the electron is free.

© 1999 S.O. Kasap, Optoelectronics (Prentice Hall)

Figure 3.1

3.1 Energy Bands (Hydrogen Atom)

\[
\begin{align*}
1s^2 & \quad 2s^2 \quad 2p^6 \quad 3s^2 \quad 3p^6 \quad 3d^{10} \quad 4s^2 \quad 4p^6 \quad 4d^{10} \\
\end{align*}
\]

Cohesive Bonded Diamond Crystal Group

- Carbon \( Z = 6 \)
- Silicon \( Z = 14 \)
- Ge \( Z = 32 \)
(a) A simplified two dimensional view of a region of the Si crystal showing covalent bonds. (b) The energy band diagram of electrons in the Si crystal at absolute zero of temperature.

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

Figure 3.2

\[ kT \sim 25.9 \text{ meV} \]

\[ h\nu (\text{eV}) \approx \frac{1.234}{\lambda (\mu\text{m})} \]
(a) A photon with an energy greater than $E_g$ can excite an electron from the VB to the CB.
(b) Each line between Si-Si atoms is a valence electron in a bond. When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created.

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

**Figure 3.3**
(a) Energy band diagram. (b) Density of states (number of states per unit energy per unit volume). (c) Fermi-Dirac probability function (probability of occupancy of a state). (d) The product of \( g(E) \) and \( f(E) \) is the energy density of electrons in the CB (number of electrons per unit energy per unit volume). The area under \( n_E(E) \) vs. \( E \) is the electron concentration.

\[ n_p = N_n^\ast \cdot \frac{(E_c - E_F)}{E_0} = E_g \]

\[ n(E) = \begin{cases} 0 & \text{in conduction band} \\ \int_{E_c}^{E_c + \Delta E} g(E) \cdot dE \cdot f(E) = N_c \exp \left[- \frac{E_c - E_F}{kT} \right] & \text{in conduction band} \end{cases} \]

\[ p(E) = \begin{cases} 0 & \text{in valence band} \\ \int_{E_V}^{E_V + \Delta E} g(E) \cdot dE \cdot (1-f(E)) = N_v \exp \left[- \frac{E_F - E_V}{kT} \right] & \text{in valence band} \end{cases} \]

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

Figure 3.4
(a) The four valence electrons of As allow it to bond just like Si but the fifth electron is left orbiting the As site. The energy required to release to free fifth-electron into the CB is very small.

(b) Energy band diagram for an n-type Si doped with 1 ppm As. There are donor energy levels just below $E_c$ around As⁺ sites.

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

**Figure 3.5**

As (Group 5) 1 extra electron compared to the host crystal atoms.

As⁺ and its 5 th free electron behave like a hydrogen atom with its energy levels.

$$E = -\frac{13.6 \text{ eV}}{n^2}$$

⇒ At room temp. As atom gets ionized (loses its 5 th electron to xtal)

* The impurity, to become an active donor or acceptor, has to be a substitutional atom. Otherwise, its electrons will not interact with the lattice, resulting in no electrical effect.
(a) Boron doped Si crystal. B has only three valence electrons. When it substitutes for a Si atom one of its bonds has an electron missing and therefore a hole. (b) Energy band diagram for a $p$-type Si doped with 1 ppm B. There are acceptor energy levels just above $E_v$ around B$^-$ sites. These acceptor levels accept electrons from the VB and therefore create holes in the VB.

© 1999 S.O. Kasap, Optoelectronics (Prentice Hall)

Figure 3.6
(a) Boron doped Si crystal. B has only three valence electrons. When it substitutes for a Si atom one of its bonds has an electron missing and therefore a hole. (b) Energy band diagram for a p-type Si doped with 1 ppm B. There are acceptor energy levels just above $E_v$ around B$^-$ sites. These acceptor levels accept electrons from the VB and therefore create holes in the VB.

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

Figure 3.6
Energy band diagrams for (a) intrinsic (b) $n$-type and (c) $p$-type semiconductors. In all cases, $np = n_i^2$. Note that donor and acceptor energy levels are not shown.

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

Figure 3.7

$$n = N_c e^{-\frac{E_g-E_F}{kT}}$$

where $n.p = n_i^2$ from $E_F$.

**$n$-Type**

$$n \approx (N_d - N_a) \Rightarrow p = \frac{n_i^2}{n_i} \ll n_i$$

**$p$-Type**

$$p = (N_a - N_d) = n = \frac{n_i^2}{p} \ll n_i$$

$$p.n = N_v e^{-\frac{E_v-E_F}{kT}}$$

$$p.n = N_v e^{-\frac{E_g-E_v}{kT}} = n_i^2$$

**log scale**

$p \gg n$
(a) Degenerate $n$-type semiconductor. Large number of donors form a band that overlaps the CB. (b) Degenerate $p$-type semiconductor.

© 1999 S.O. Kasap, Optoelectronics (Prentice Hall)

**Figure 3.8**

Extremely heavily doped

$\begin{align*}
N^+ \text{Type } & n > N_c \Rightarrow E_F > E_c \\
P^+ \text{ Type} & p > N_v \Rightarrow E_F < E_v
\end{align*}$

+ Impurity band formation $\Rightarrow$

$E_F$ shrinkage compared to normal doping

(This results in saturation of $\beta$ one can get by increasing emitter doping)

$\beta$

$+$ Bandgap shrinking

$N_{\text{Emitter}}$

$N_{\text{Base}}$
Energy band diagram of an n-type semiconductor connected to a voltage supply of $V$ volts. The whole energy diagram tilts because the electron now has an electrostatic potential energy as well.

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

Figure 3.9
The electron potential energy ($PE$), $V(x)$, inside the crystal is periodic with the same periodicity as that of the crystal, $a$. Far away outside the crystal, by choice, $V = 0$ (the electron is free and $PE = 0$).

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

Figure 3.10

$m \rightarrow m^+$

Particle $\equiv$ wave which obeys Schrödinger’s eqn.:

\[
\frac{d^2 u}{dx^2} + \frac{2m}{\hbar^2} [E - V(x)] = 0 \quad \Rightarrow \text{solutions are sinusoidal or } e^{i(kx - \omega t)}
\]

\[ kL = \pi \cdot n \]

\[ k^2 = \frac{2m}{\hbar^2} [E - V] \Rightarrow E = \frac{n^2 \hbar^2}{2m} \alpha n^2 \]
The $E$-$k$ diagram of a direct bandgap semiconductor such as GaAs. The $E$-$k$ curve consists of many discrete points with each point corresponding to a possible state, wavefunction $\psi_k(x)$, that is allowed to exist in the crystal. The points are so close that we normally draw the $E$-$k$ relationship as a continuous curve. In the energy range $E_v$ to $E_c$ there are no points ($\psi_k(x)$ solutions).

© 1999 S.O. Kasap, Optoelectronics (Prentice Hall)

Figure 3.11

Quantum Mech.

$p$ (momentum) = $\hbar k$

Light (photon) can exert energy but little momentum.

A vertical transition on the $E$-$k$ diagram
(a) In GaAs the minimum of the CB is directly above the maximum of the VB. GaAs is therefore a direct bandgap semiconductor. (b) In Si, the minimum of the CB is displaced from the maximum of the VB and Si is an indirect bandgap semiconductor. (c) Recombination of an electron and a hole in Si involves a recombination center.

© 1999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

**Figure 3.12**

- Indirect Band
- Electron-hole generation / recombination needs help from phonons = a quantum of vibrational energy (acoustic)
- G-R processes are very low probability (slow) in Indirect Band Materials.
- Good for BJT
- Bad for LED, Laser
World Record for Silicon Light

By Dr. Gernot S. Pomrenke, AFRL's Air Force Office of Scientific Research

The world’s third-largest semiconductor manufacturer discovered groundbreaking technology that allows silicon-based light emitters to match the efficiency of traditional light-emitting compound semiconductor materials such as gallium arsenide. The creative origins of STMicroelectronics’ new technology, which sets a world record for quantum efficiency, are linked to important AFRL-sponsored studies conducted jointly in 1993 by the laboratory and the Massachusetts Institute of Technology. The team’s research in rare-earth-doped, light-emitting diodes inspired STMicroelectronics’ recent innovation.

The new technology is based on a structure in which researchers implanted ions of rare earth metals, such as erbium or cerium, in a layer of silicon-rich oxide. The technology opens up many potential applications in which optical and electrical functions reside on a single silicon chip. Although silicon is ideal for building computer memory, microprocessors, and other complex circuits, it is not an efficient light emitter. “The ability to combine optical and electronic processing on the same chip presents enormous opportunities,” said Mr. Gian Guido Rizzato, STMicroelectronics’ director of corporate technology research and development. “We have identified a number of promising applications, and key manufacturing issues have already been solved so the technology can be rapidly moved into production.” One of the first applications of the new technology will be to build power control devices in which the control circuitry is electrically isolated from the power switching transistors. Currently, engineers can achieve electrical isolation—mandatory in many applications for safety reasons—only by using external devices such as relays, transformers, or discrete optocouplers.

In an extension of this technology, STMicroelectronics patented a novel structure in which two circuits, built on the same chip but electrically separated from each other by insulating silicon dioxide, communicate via optical signals using integrated silicon light emitters and detectors. Devices based on this design will have a number of important uses, including motor controls, power supplies, solid-state relays, and similar applications in which the power circuit operates at much higher voltages than the control circuit.
Before and After the Junction is Formed

**P-Type**

Before

- **Acceptors** (Group III)
- **Holes** (minority)
- **Electrons** (majority)

**N-Type**

- **Donors** (Group V)
- **Holes** (minority)
- **Electrons** (majority)

1. **Space Charge Neutrality**

\[ N_A + n = P^+ \]

\[ N_D + P = n^- \]

2. **Thermal Equilibrium**

\[ P \cdot n = n_i^2 \]

\[ P_{no} \cdot n_{no} = n_i^2 \]

for **P-Type side** in equilibrium

for **N-Type side**

3. \[ P_{po} = \frac{N_A}{2} + \sqrt{\left(\frac{N_A}{2}\right)^2 + n_i^2} \]

\[ n_{no} = \frac{N_D}{2} + \sqrt{\left(\frac{N_D}{2}\right)^2 + n_i^2} \]

for **N_A \gg n_i**

for **N_D \gg n_i**

4. **Fermi Level**

\[ n = N_c e^{-\frac{(E_F - E_c)}{kT}} \]

\[ p = N_v e^{-\frac{(E_F - E_v)}{kT}} \]

Vacuum level closer

\[ E_c \rightarrow E_{Fp} \]

\[ E_v \rightarrow E_{Fn} \]

\[ E_i = \cdot \cdot \cdot \]
After the Contact

1. Mobile charges get attracted to the other side.
2. Mobile charge transfer takes place; results in recombination and annihilation of charges (electrons/holes).
3. The fixed (locally) charge left behind remains uncompensated. ⇒ A net dipole charge left behind.
4. Space charge density

\[ \phi = \varepsilon \left\{ \begin{array}{ll} +N_d & \text{N-Type side} \\ -N_A & \text{P-Type side} \end{array} \right. \]

Maxwell's eqn.:

\[ \nabla \cdot D = \phi \]

⇒ \( \varepsilon \nabla \cdot E = \phi \)

\[ -\nabla V = E \]

\[ \frac{\partial^2 V}{\partial x^2} = -\frac{\phi}{\varepsilon} \]

Integrate and solve for the unknown.

\[ V(x) = -\int E(x) \, dx \]

\[ E(x) = -\int \left( \frac{\partial V}{\partial x} \right) \, dx \]

DoP \$\$

Built-in potential if no external voltage is applied.
Properties of the pn junction.

\[ \int_{\text{const}} \, dx = x \]
\[ \int x \, dx = \frac{x^2}{2} \]

**Figure 3.13**

\[ \Delta V = \int \Delta V_{N-type} = \frac{e \cdot N_a}{2e} W_n^2 \]
\[ \Delta V_{P-type} = \frac{e \cdot N_a}{2e} W_p^2 \]
potential drop in P-side

$$\Delta V_p = -\int\frac{(eN_A)dx}{\epsilon}$$

$$\Delta V_p = \frac{eN_A W_p^2}{2\epsilon}$$

potential drop in N-side

$$\Delta V_n = -\int\frac{eN_0 dx}{\epsilon}$$

$$\Delta V_n = \frac{eN_0 W_n^2}{2\epsilon}$$

Since dipole has equal and opposite charge across junction

$$Q_{P-side} = eN_A W_p$$

$$Q_{N-side} = eN_D W_n$$

Conclusion 1:

$$\frac{W_p}{N_A} = \frac{(N_A)_{P-side}}{(N_D)_{N-side}}$$

\[\therefore \text{For a one-sided junction like (P'}^+\text{-N)}\]

$$N_A \gg N_D \implies W_p \ll W_n$$

i.e. the space charge penetrates deeper in the lightly doped side of the junction.

Conclusion 2:

$$\Delta V_p = \frac{eN_A W_p^2}{2\epsilon} = \left(\frac{eN_A}{eN_D}\right) W_p$$

\[\therefore \text{For the same one-sided (P'}^+\text{-N) junction}\]

$$N_A \gg N_D \implies W_p \ll W_n \text{ and therefore}$$

$$\Delta V_p \ll \Delta V_n$$

\[\therefore \text{i.e. most of the potential drop occurs in the lightly doped side}\]

Conclusion 3:

$$\Delta V \approx \Delta V_n = \frac{eN_0 W_n^2}{2\epsilon} \implies W_n \approx \sqrt{\frac{2\epsilon \Delta V}{eN}}$$

for P'}^+\text{-N junction
Boltzmann Relationship

In a neutral semiconductor in equilibrium (no applied voltage) or disturbance,

\[ J = J_n + J_p = 0 \]

\[ 0 = 0 \]

\[ J_n = e \mu_n n E(x) + e D_n \frac{2 \partial n}{\partial x} = 0 \implies E(x) = -\frac{D_n}{\mu_n} \frac{2 \partial n}{\partial x} \]

\[ J_p = e \mu_p p E(x) - e D_p \frac{2 \partial p}{\partial x} = 0 \implies E(x) = \frac{D_p}{\mu_p} \frac{2 \partial p}{\partial x} \]

Integrating either and using Einstein's Relationship

\[ \frac{D}{\mu} = kT/q \]

\[ \Delta V_{21} = -\int^2_1 E(x) dx = -\int^2_1 \left(-\frac{kT}{q} \frac{2 \partial n}{\partial x} \right) dx = +\frac{kT}{q} \left( \ln \frac{n_2}{n_1} - \ln \frac{n_1}{n_2} \right) = +\frac{kT}{q} \ln \left( \frac{n_2}{n_1} \right) \]

\[ \frac{n_2}{n_1} = \exp \left( \frac{\Delta V_{21}}{kT} \right) \]

\[ \frac{p_2}{p_1} = \exp \left( -\frac{\Delta V_{21}}{kT} \right) \]

These equations apply when the current is small as well as when it is zero.

Apply above to a P-N junction in equilibrium

\[ \Delta V_{21} = V_0 = \text{Built-in potential} \]

\[ p_2 = n_0, \quad p_1 = p_{po} \]

\[ V_0 = \frac{kT}{q} \ln \left( \frac{p_{po}}{n_0} \right) \]

\[ p_{po} = \frac{n_2}{n_1} n_0 \]
\[ V_0 = \frac{kT}{q} \ln \left( \frac{n_{eq} p_{eq}}{n_0^2} \right) \]

Since \( p_{eq} = N_A \) and \( n_{eq} = N_0 \),

\[ V_0 \approx \frac{kT}{q} \ln \left( \frac{N_A \cdot N_0}{n_0^2} \right) \]

**Built-in potential after contact**

Energy Band View:

- \( E_c \): Vacuum
- \( E_F \): Electrons
- \( E_F \): Holes
- \( E_v \): Before Contact
- \( A.C. \): After Contact

\[ E_c - E_F = eV_0 + (E_c - E_{F_n}) \quad \Rightarrow \quad eV_0 = E_{F_n} - E_F \]

When a bias voltage, \( V \) is applied

\[ \Delta V = V_0 - V \] if \( V \)'s (+) terminal is tied to P-Side (FORWARD BIASING)

Therefore \( \Delta V \) is reduced.
Forward biased $pn$ junction and the injection of minority carriers (a) Carrier concentration profiles across the device under forward bias. (b) The hole potential energy with and without an applied bias. $W$ is the width of the SCL with forward bias.

© 1999 S.O. Kasap, Optoelectronics (Prentice Hall)

**Figure 3.14**

Assuming $V_{\text{Applied}}$ is not excessive, i.e., injection of carriers across the energy-barrier lowered junction ($V_0 \rightarrow V_0 - V_{\text{Applied}}$) does not change the majority concentrations.

\[
\Delta V_{21} = V_0 - V_{\text{Applied}}
\]

\[
P_2 = P_n(0), \quad P_1 \equiv P_{po} = \text{unaffected}
\]

\[
P_n(0) = \frac{P_n(0)}{P_{po}} \exp \left(-\frac{V_0 - V_{\text{Applied}}}{kT}\right) = \exp \left(-\frac{V_0}{kT}\right) \exp \left(\frac{V_{\text{Applied}}}{kT}\right)
\]

\[
P_n(0) = P_{po} \exp \left(-\frac{V_0}{kT}\right) \exp \left(\frac{V_{\text{Applied}}}{kT}\right) = P_{no} e^{\left(-\frac{V_{\text{Applied}}}{kT}\right)}
\]

\[
\text{Pno r before } V_{\text{Applied}}
\]